International Journal of Theoretical Physics, Vol. 29, No. 3, 1990

Nonlocal Stochastic Quantization of Scalar Electrodynamics

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Received November 3, 1989

Quantization of the electromagnetic interactions of scalar charged particles is considered within the stochastic Langevin and Schwinger–Dyson equations with nonlocal white noise. Fulfillment of the gauge-invariant condition in such a scheme is studied in detail. Matrix elements of the vacuum polarization and self-energy diagrams of the scalar electrodynamics are calculated explicitly, which reduce to usual nonlocal scalar electrodynamic results.

1. INTRODUCTION

Within the framework of stochastic quantization (Parisi and Wu, 1981), methods of nonequilibrium statistical mechanics have been used to study different theoretical field models (for example, see Migdal, 1986). This alternative method of quantization of physical systems to the usual Hamiltonian, path integral, and action formulations has given birth to a number of new ideas and to the understanding of many problems of field theory. In particular, these developments include Zwanziger's gauge fixing (Zwanziger, 1981; Floratos *et al.*, 1984), large-N quenching and large-N master fields (Alfaro and Sakita, 1983; Greensite and Halpern, 1983), stochastic stabilization (Greensite and Halpern, 1984), stochastic regularization (Bern *et al.*, 1987*a,b*; Niemi and Wijewardhana, 1982), and numerical applications of the Langevin equation in lattice gauge theory (Hamber and Heller, 1984; Batrouni *et al.*, 1985). Moreover, the stochastic scheme of construction of quantum field theory may be technically superior and useful for the quantization of gravity (Halpern, 1987; Chan and Halpern, 1987) and for nonperturbative analysis (Doering, 1985).

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In a previous paper (Dineykhan and Namsrai, 1988) we studied the regularization problem of the Langevin and Schwinger-Dyson equations within nonlocal quantum field theory (Efimov, 1977, 1985).

The basic idea of our prescription consisted in considering, instead of the usual differential equations of the stochastic theory, their modified versions with nonlocal white noise

$$\eta(x, t) \to \eta_{\text{nonloc}}(x, t) = \int (dy) K(x-y)\eta(y, t), \qquad (dy) = d^4y \tag{1}$$

Here $K(x) = K(\Box l^2)\delta^{(4)}(x)$ is a nonlocal generalized function (Efimov, 1977, 1985), and t and l are parameters of the theory we call the "fifth time" and the fundamental length, respectively. The local white noise in (1) satisfies the condition

$$\langle \eta(x, t)\eta(y, t')\rangle_\eta = 2\delta^{(4)}(x-y)\delta(t-t')$$

The nonlocal white noise (1) plays a double role in the stochastic quantization method; it controls the quantum behavior of a physical system and at the same time it carries nonlocality in stochastic equations, i.e., it makes the theory finite in each order of the perturbation series of the coupling constant. This method was applied to the investigation of scalar and gauge fields and also to the calculation of photon and gluon masses, which turn out to be zero (Dineykhan and Namsrai, 1988). The present paper continues our previous work and is devoted to the construction of gauge-invariant finite scalar electrodynamics by means of a stochastic quantization scheme with the nonlocal white noise (1), and an outline of which is as follows. In Section 2 it is shown that in our approach the gauge-invariant condition is fulfilled and in calculations of gauge-invariant quantities a dependence on a gauge-fixing parameter does not appear. Further, we construct the electromagnetic interaction of scalar particles in the Schwinger-Dyson formalism. Sections 4 and 5 are devoted to the calculation of the vacuum polarization and the self-energy diagrams for scalar particles.

2. THE LANGEVIN EQUATION IN GAUGE THEORIES AND GAUGE-FIXING PROCEDURE

The basic equations of the stochastic quantization (Parisi and Wu, 1981; Bern *et al.*, 1987*a*) are the Langevin and Schwinger-Dyson equations. These define the behavior of field functions and their dependence on the white noise. In particular, the Langevin equation for a gauge field $A^a_\mu(x, t)$ takes the form

$$\frac{\partial A^a_\mu(x, t)}{\partial t} = -\frac{\delta S}{\delta A^a_\mu} + D^{ab}_\mu \Lambda^b + \int (dy) K^{ab}_{xy}(\Delta)\eta^b_\mu(y, t) \tag{2}$$

where

$$S=-\frac{1}{4}\int(dx)F^a_{\mu\nu}(x)F^a_{\mu\nu}(x)$$

is the standard form of the gauge field action and $F^a_{\mu\nu}(x)$ is the stress tensor of the Yang–Mills field:

$$F^a_{\mu\nu}(x)=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+gf^{abc}A^b_\mu(x)A^c_\nu(x)$$

The second term on the right-hand side of (2) is called the Zwanziger (1981) term, which determines the gauge fixing and has the form

$$D^{ab}_\mu\Lambda^b=\frac{1}{\alpha}[\delta^{ab}\partial_\mu-gf^{abc}A^c_\mu(x)]\partial_\nu A^b_\nu(x)$$

where α is the gauge-fixing parameter. In our case, the white noise is distributed by means of the nonlocal generalized function $K^{ab}_{xy}(\Box)$, the Fourier transform of which is an entire analytic function of the variable $x=p^2l^2$ with a finite order of growth $\infty>\rho\geq 1/2$ and decreases rapidly enough when $z\to\infty$ [for details see Efimov (1977)]. For equation (2) the nonlocal distribution $K^{ab}_{xy}(\Delta)$ is a function of the covariant Laplacian

$$\Delta^{ab}_{xy}=\int(dz)D^{ac}_\mu(z)\delta^d(z-x)D^{cb}_\mu(z)\delta^{(d)}(z-y) \qquad (3)$$

where

$$(dz)=d^dz$$

$$D^{ab}_\mu(x)=\delta^{ab}\partial_\mu+gf^{acb}A^c_\mu(x)$$

and d is the dimension of space-time. This definition generalizes the usual Laplacian $\Box_{xy}$,

$$\Box_{xy}=\int(dz)\partial^z_\mu\delta^d(x-z)\partial^z_\mu\delta^d(z-y)$$

From (3) it is easily seen that

$$K^{ab}_{xy}(\Delta)=K^{ba}_{yx}(\Delta)$$

In the weak coupling limit the nonlocal distribution $K^{ab}_{xy}(\Delta)$ may be decomposed by a power of the coupling constant g:

$$\begin{aligned}K^{ab}_{xy}(\Delta)=K^{ab}_{xy}(\Box)&+\tfrac{1}{2}g[K^{(1)}(\Box)\Gamma_1H(\Box)+H(\Box)\Gamma_1K^{(1)}(\Box)]^{ab}_{xy}\\&+\tfrac{1}{2}g^2[K^{(1)}(\Box)\Gamma_2H(\Box)+H(\Box)\Gamma_2K^{(1)}(\Box)]^{ab}_{xy}\\&+\tfrac{1}{6}g^2[K^{(2)}(\Box)\Gamma_1H(\Box)\Gamma_1H(\Box)\\&+H(\Box)\Gamma_1K^{(2)}(\Box)\Gamma_1H(\Box)+H(\Box)\Gamma_1H(\Box)\Gamma_1K^{(2)}(\Box)]^{ab}_{xy} \qquad (4)\end{aligned}$$

Here the Fourier transforms of generalized functions $K^{(i)}(\Box)$ are given by [for details, see Dineykhan and Namsrai (1988)]

$$K^{(1)}(p^2l^2)=\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{w(\xi)}{\sin \pi\xi}\,\xi(p^2l^2)^{\xi} \qquad (0<\beta<1)$$

$$K^{(2)}(p^2l^2)=\frac{1}{2i}\int_{\beta+i\infty}^{\beta-i\infty} d\xi \frac{w(\xi)}{\sin \pi\xi}\,\xi(\xi-1)(p^2l^2)^{\xi}$$

and for the operator $H_{xy}(\Box)=(\Box_x l^2)^{-1}\delta^{(d)}(x-y)$ we have

$$H_{xy}(\Box)=-\int (dp)\, e^{-ip(x-y)}H(p^2l^2)$$

$$H(p^2l^2)=p^{-2}l^{-2}$$

The vertex functions Γ_1 and Γ_2 in (4) are given by

$$(\Gamma_1)_{xy}^{ab}=l^2f^{abc}[A_\mu^c(x)\partial_\mu+\partial_\mu A_\mu^c(x)]\delta^{(d)}(x-y)$$

$$(\Gamma_2)_{xy}^{ab}=l^2f^{acn}f^{cbm}A_\mu^n(x)A_\mu^m(x)\delta^{(d)}(x-y)$$

It should be noted that equation (2) is invariant under the following local gauge transformations:

$$\begin{aligned}\dot{A}_\mu^a(x,t)&\to\Omega^{ab}(x)\dot{A}_\mu^b(x,t)\\ \eta_\mu^a(x,t)&\to\Omega^{ab}(x)\eta_\mu^b(x,t)\\ K_{xy}^{ab}(\Delta)&\to\Omega^{aa'}(x)\Omega^{bb'}(y)K_{xy}^{a'b'}(\Delta)\end{aligned} \qquad (5)$$

where $\Omega^{ab}(x)\subset SO(N^2-1)$ is an adjoint representation of $SU(N)$.

As shown by Bern *et al.* (1987*a*), from the Langevin equation (2) without the Zwanziger term one can easily obtain the Schwinger–Dyson equation. After some standard calculations (Dineykhan and Namsrai, 1988) from equation (2) we have

$$\frac{dF[A]}{dt}=-LF[A]$$

where

$$L=-\int (dx)\left\{\int (dz)(dy)\left[K_{zy}^{bc}(\Delta)\frac{\delta K_{zx}^{ba}(\Delta)}{\delta A_\mu^c(y)} + K_{zy}^{bc}(\Delta)K_{zx}^{ba}(\Delta)\frac{\delta}{\delta A_\mu^c(y)}\right]-\frac{\delta S}{\delta A_\mu^a(x)}\right\}\frac{\delta}{\delta A_\mu^a(x)} \qquad (6)$$

and $F[A]$ is a gauge-invariant functional satisfying the condition

$$G^a(x)F[A]=0$$

for the generator of the gauge transformation:

$$G^a(x)=-D_\mu^{ab}(x)\frac{\delta}{\delta A_\mu^b(x)} \tag{7}$$

for which the commutation relation is defined as

$$[G^a(x), G^b(y)]=gf^{abc}\delta^{(d)}(x-y)G^c(x)$$

Now we show that the following gauge condition for L is fulfilled:

$$[G^a(x), L]=0 \tag{8a}$$

In our case, according to Efimov (1977), the generalized function $K_{xy}^{ab}(\Delta)$ may be represented in the form

$$K_{xy}^{ab}(\Delta)=\sum_{n=0}^{\infty}\frac{C_n}{(2n)!}(\Delta^n)_{xy}^{ab}$$

The space-time properties of $K_{xy}^{ab}(\Delta)$ are defined by a concrete form of the coefficients C_n [generally, they are complex quantities; for details, see Efimov (1977)]. The product of generalized functions $K_{xy}(\Box)$ may be understood as a contraction operation only. For example,

$$K_{xy}^2(\Box)=\int (dz)K_{xz}(\Box)K_{zy}(\Box)$$

or

$$\Box_{xy}^2=\int (dz)\Box_{xz}\Box_{zy}$$

so that

$$(\Delta^n)_{xy}^{ab}=\int (dz_1)\cdots\int (dz_{n-1})\Delta_{xz_1}^{ac_1}\Delta_{z_1z_2}^{c_1c_2}\cdots\Delta_{z_{n-1}y}^{c_{n-1}b}$$

Using the explicit form of (3) and carrying out integrations over (dz_j), we have

$$(\Delta^n)_{xy}^{ab}=\Delta_x^{ac_1}\Delta_x^{c_1c_2}\cdots\Delta_x^{c_{n-1}b}\delta^{(d)}(x-y)=(\Delta_x^n)^{ab}\delta^{(d)}(x-y)$$

where

$$\Delta_x^{ab}=D_\mu^{ac}(x)D_\mu^{cb}(x)$$

Thus, the distribution of the white noise is written as

$$K_{xy}^{ab}(\Delta)=\sum_{n=0}^{\infty}\frac{C_n}{(2n)!}(\Delta_x^n)^{ab}\delta^{(d)}(x-y) \tag{9}$$

Let us consider the particular case when $n=0$, i.e.,

$$K_{xy}^{ab}=\delta^{ab}\delta^{(d)}(x-y)$$

Then from (6) we have

$$L_0=-\int(dx)\left[\frac{\delta}{\delta A_\mu^a(x)}-\frac{\delta S}{\delta A_\mu^a(x)}\right]\frac{\delta}{\delta A_\mu^a(x)}$$

Taking into account the equality

$$\frac{\delta S}{\delta A_\nu^a(x)}=-D_\mu^{ab}(x)F_{\mu\nu}^b(x)$$

and (6), and after some simplifications, we get for $[G^a(x), L_0]$

$$\begin{aligned}[G^a(x), L_0]=\int(dy)\Bigg\{&D_\mu^{ab}(x)\frac{\delta D_\rho^{nm}(y)}{\delta A_\mu^b(x)}F_{\rho\nu}^m(y)\frac{\delta}{\delta A_\nu^n(y)}\\&+D_\mu^{ab}(x)D_\rho^{nm}(y)\frac{\delta F_{\rho\nu}^m(y)}{\delta A_\mu^b(x)}\frac{\delta}{\delta A_\nu^n(y)}\\&-D_\rho^{nm}(y)F_{\rho\nu}^m(y)\frac{\delta D^{ab}(y)}{\delta A_\nu^n(y)}\frac{\delta}{\delta A_\mu^b(x)}\\&-2\frac{\delta D_\mu^{ab}(x)}{\delta A_\nu^n(y)}\frac{\delta}{\delta A_\nu^n(y)}\frac{\delta}{\delta A_\mu^b(x)}\Bigg\}\end{aligned}\tag{8b}$$

Further, we use the functional differential form

$$\delta D_\rho^{nm}(y)/\delta A_\mu^b(x)=gf^{nbm}\delta_{\rho\mu}\delta^{(d)}(x-y)$$

and carry out simple calculations for each term in (8b). The result is

$$\begin{aligned}\int(dy)\Bigg[&D_\mu^{ab}(x)-\frac{\delta D_\rho^{nm}(y)}{\delta A_\mu^b(x)}F_{\rho\nu}^m(x)\frac{\delta}{\delta A_\nu^n(x)}\\&-D_\rho^{nm}(y)F_{\rho\nu}^m(y)\frac{\delta D_\mu^{ab}(x)}{\delta A_\nu^n(y)}\frac{\delta}{\delta A_\mu^b(x)}\Bigg]\\&=-f^{ncm}f^{mab}g^2\cdot A_\mu^c(x)F_{\mu\nu}^b(x)\frac{\delta}{\delta A_\nu^n(x)}\end{aligned}$$

$$D_\mu^{ab}(x)\int(dy)D_\rho^{nm}(y)[\delta F_{\rho\nu}^m(y)/\delta A_\mu^b(x)][\delta/\delta A_\nu^n(y)]$$

$$=g^2f^{ncm}f^{maj}A_\mu^c(x)F_{\mu\nu}^j(x)\frac{\delta}{\delta A_\nu^n(x)}$$

and

$$\int (dy)\frac{\delta D_\mu^{ab}(x)}{\delta A_\nu^n(y)}\frac{\delta}{\delta A_\nu^n(y)}\frac{\delta}{\delta A_\mu^b(y)} = gf^{anm}\frac{\delta^2}{\delta A_\mu^n(x)\delta A_\mu^m(x)}$$

Taking the sum of all these terms, it is easily seen that

$$[G^a(x), L_0] = gf^{anm}\frac{\delta^2}{\delta A_\mu^n(x)\delta A_\mu^m(x)} \equiv 0$$

Now let us consider the case when $g=0$. From (4) and (9) it follows that

$$K_{xy}^{ab}(\Box) = \sum_{n=0}^{\infty}\frac{c_n}{(2n)!}\Box_x^{n(d)}(x-y)\delta^{ab}$$

In this approximation the expression (6) acquires the form

$$L_1 = -\int (dx)\left[\iint (dz)(dy)K_{zy}^{cb}(\Box)K_{zy}^{ba}(\Box)\frac{\delta}{\delta A_\mu^c(y)} - \frac{\delta S}{\delta A_\mu^b(x)}\right]\frac{\delta}{\delta A_\mu^a(x)}$$

After some analogous calculations carried out above for the commutator $[G^a(x), L_0]$, we get

$$[G^a(x), L_1] = \iint (dy)(dz)K_{xz}^{mn}(\Box)K_{zy}^{nb}(\Box)f^{amc}\frac{\delta^2}{\delta A_\mu^c(y)\delta A_\mu^b(x)}$$

If we take into account the following identity for $K(\Box)$ (Efimov, 1977),

$$\int (dz)K_{xz}^{mn}(\Box)K_{zy}^{nb}(\Box) = \sum_j a_j\Box_x^j\delta^{(d)}(x-y)\delta^{mb}$$

then we see immediately that

$$[G^a(x), L_1] = \sum_j a_j\Box^j f^{abc}\frac{\delta^2}{\delta A_\mu^b(x)\delta A_\mu^c(x)} = 0$$

where $a_j = c_j/(2j)!$.

Let us consider the general case, when the nonlocal white noise is defined by the distribution (9). From the decomposition (4) it is seen that we have already shown the fulfillment of the gauge-invariant condition (8) for some of its components. By an analogous procedure as followed above for two particular cases, one can verify that this condition is also satisfied. However, the intermediate expressions are very long and cumbersome, and therefore we do not give their explicit form as in the previous cases.

As mentioned by Migdal (1986), in the stochastic quantization scheme terms growing over t are mutually canceled in each order of the perturbation series for any gauge-invariant quantities and therefore in the Langevin equation of the gauge theory one does not need to introduce gauge-fixing terms. It is also possible (Zwanziger, 1981) to transform the Langevin equation in order to make these terms vanish, i.e., if the condition (8) is fulfilled, then at the end of the calculations of gauge-invariant quantities the dependence on the parameter α falls out. In this way, any fields of the type of ghosts are not needed.

In the stochastic quantization scheme with nonlocal white noise the gauge-invariant condition (8) is also fulfilled, and therefore calculation of any gauge-invariant quantities for this nonlocal theory does not require the introduction of intermediate ghost fields.

In the next sections we show that gauge-invariant quantities do not depend on the gauge-fixing parameter. To do this, we consider electromagnetic interaction of charged scalar particles within the stochastic quantization with nonlocal white noise.

3. THE SCHWINGER–DYSON FORMALISM AND ELECTROMAGNETIC INTERACTION OF SCALAR PARTICLES

The Langevin equations for electromagnetic field $A_\mu(x, t)$ and charged scalar particle fields $\varphi(x, t)$ and $\varphi^*(x, t)$ are written as follows:

$$\frac{dA_\mu(x, t)}{dt} = -\frac{\delta S}{\delta A_\mu}(x, t) + \partial_\mu \Lambda(y, t) + \int (dy) K_{xy}(\Box) \eta_\mu(y, t) \tag{10a}$$

$$\frac{d\varphi(x, t)}{dt} = -\frac{\delta S}{\delta \varphi^*}(x, t) + ie\varphi(x, t)\Lambda(x, t) + \int (dy) K_{xy}(\Delta) \eta_\mu(y, t) \tag{10b}$$

$$\frac{d\varphi^*(x, t)}{dt} = -\frac{\delta S}{\delta \varphi}(x, t) - ie\varphi^*(x, t)\Lambda(x, t) + \int (dy) K_{xy}(\Delta^*) \eta^*(y, t) \tag{10c}$$

where S is the usual action of electromagnetic interaction of the scalar charged particle:

$$S = \int (dx)\{\tfrac{1}{4}F_{\mu\nu}(x)F_{\mu\nu}(x) + |(\partial_\mu - ieA_\mu)\varphi|^2 + m^2|\varphi|^2\}$$

and $F_{\mu\nu}(x)$ is the stress tensor of the electromagnetic field

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) = \partial_\nu A_\mu(x)$$

The second term on the right-hand side of (10a) and (10c) depends on the Zwanziger gauge-fixing procedure (Zwanziger, 1981) and is defined as

$$\Lambda(x, t) = \frac{1}{\alpha}\partial_\mu A_\mu(x)$$

Local white noises $\eta_\mu(x, t)$ and $\eta(x, t)$, $\eta^*(x, t)$ of the electromagnetic and charged scalar fields in equations (10a)-(10c) satisfy the usual relations

$$\langle \eta_\mu(x, t)\eta_\nu(y, t')\rangle_\eta = 2\delta_{\mu\nu}\delta(t-t')\delta^{(d)}(x-y)$$
$$\langle \eta^*(x, t)\eta(y, t')\rangle_\eta = 2\delta(t-t')\delta^{(d)}(x-y)$$
$$\langle \eta(x, t)\eta(y, t')\rangle_\eta = \langle \eta^*(x, t)\eta^*(y, t')\rangle_\eta = 0$$

The corresponding distributions $K_{xy}(\Box)$ amd $K_{xy}(\Delta)$, $K_{xy}(\Delta^*)$ are generalized functions, the Fourier transforms of which are entire analytic functions of the variable $z = p^2 l^2$. These functions are usually called form factors of the theory. In equations (10a)-(10c) we have used the following notation, as in (3):

$$\Delta_{xy} = \int (dz) D_\mu(z)\delta^{(d)}(x-z) D_\mu(z)\delta^{(d)}(z-y)$$

$$\Delta^*_{xy} = \int (dz) D^*_\mu(z)\delta^{d}(x-z) D^*_\mu(z)\delta^{(d)}(z-y)$$

where

$$D_\mu(x) = \partial_\mu - ieA_\mu(x)$$
$$D^*_\mu(x) = \partial_\mu + ieA_\mu(x)$$

The Langevin equations (10a)-(10c) are invariant under the gauge transformations

$$\dot{A}_\mu \to \dot{A}_\mu + \partial_\mu f(x, t)$$
$$\dot{\varphi} \to e^{-ief}\dot{\varphi}$$
$$\eta_\mu \to \eta_\mu + K^{-1}(\Box)\partial_\mu f$$
$$\eta \to e^{-ief}\eta$$

for any functions $f(x, t)$ and $K^{-1}(\Box)$ is the inverse operator of $K(\Box)$.

According to (10a)-(10c) and after some simple functional transformations [for details, see Dineykhan and Namsrai (1988)] the Schwinger-Dyson equation for electromagnetic and charged scalar fields may be easily obtained. Thus, for the vector field we have

$$\left\langle \int (dx)\left[-\frac{\delta S}{\delta A_\mu} + \partial_\mu \Lambda \right.\right.$$
$$\left.\left. + \int\int (dy)(dz) K_{xy}(\Box) K_{yz}(\Box) \frac{\delta}{\delta A_\mu(z)} \right] \frac{\delta F[A]}{\delta A_\mu(x)} \right\rangle = 0 \qquad (11a)$$

and for a scalar field

$$\left\langle \int (dx)\left[-\frac{\delta S}{\delta\varphi^*}+ie\varphi\Lambda + \iint (dy)(dz)K_{xy}(\Delta)K_{yz}(\Delta)\frac{\delta}{\delta\varphi^*(z)}\right]\frac{\delta F[\varphi]}{\delta\varphi(x)}\right\rangle = 0 \tag{11b}$$

and

$$\left\langle \int (dx)\left[-\frac{\delta S}{\delta\varphi}-ie\varphi^*\Lambda + \iint (dy)(dz)K_{xy}(\Delta^*)K_{yz}(\Delta^*)\frac{\delta}{\delta\varphi^*(z)}\right]\frac{\delta F[\phi]}{\delta\varphi^*(x)}\right\rangle = 0 \tag{11c}$$

Here $F[A]$ and $F[\varphi]$ are gauge-invariant functionals. By using equations (11a)-(11c), we will calculate correlation functions for electromagnetic and scalar fields. From the Schwinger–Dyson equations (11a)-(11c) it is easily seen that correlation functions for the A_μ, φ, and φ^* fields are expressed through contraction between generalized functions which may be represented in the form

$$[K^A_{xy}(\Box)]^2 = \int (dz)K_{xz}(\Box)K_{zy}(\Box)$$

$$[K^\varphi_{xy}(\Delta)]^2 = \int (dz)K_{xz}(\Delta)K_{zy}(\Delta)$$

Efimov (1977) shows that the contraction operation of entire analytic functions also give entire analytic ones. Further, we use momentum space, in which the Fourier transform of the generalized functions $K^A_{xy}(\Box)$ is written as (Efimov, 1977)

$$K^A(p^2l^2) = \sum_{n=0}^{\infty}\frac{c_n}{(2n)!}(p^2l^2)^n$$

where l has the dimension of length; we call it the fundamental length, which characterizes the scale of nonlocal interactions. In concrete calculations the Mellin representation

$$K^A(p^2l^2) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi\frac{v(\xi)}{\sin\pi\xi}(p^2l^2)^\xi, \qquad 0\le\beta\le 1 \tag{12}$$

is usually used for $K^A(p^2l^2)$. As in the case (4), for the form factor of a scalar particle we carry out the following decomposition:

$$K^\varphi_{xy}(\Delta) = K^\varphi_{xy}(\Box) + \tfrac{1}{2}e[K^{(1)}(\Box)\Gamma^e_1 H(\Box) + H(\Box)\Gamma^e_1 K^{(1)}(\Box)]_{xy} + \cdots$$

over powers of the coupling constant e. The corresponding vertices are given by

$$(\Gamma_1^e)_{xy} = -il^2[A_\mu(x)\partial_\mu + \partial_\mu A_\mu(x)]\delta^{(d)}(x-y)$$
$$(\Gamma_2^e)_{xy} = -l^2[A_\mu(x)A_\mu(x)]\delta^{(d)}(x-y)$$

Next, we restrict ourselves to order e^3 and pass to the momentum representation. In this approximation from the decomposition of $K_{xy}^\varphi(\Delta)$ and (4) one can easily see that $K_{xy}^\varphi(\Delta)$ is expressed through the functions $K_{xy}^\varphi(\Box)$, $K_{xy}^{\varphi(1)}(\Box)$, $K_{xy}^{\varphi(2)}(\Box)$, and $K_{xy}^{\varphi(3)}(\Box)$. The corresponding Fourier transforms are denoted by $V(p^2l^2)$, $V^{(1)}(p^2l^2)$, $V^{(2)}(p^2l^2)$, and $V^{(3)}(p^2l^2)$ (see Figure 2), for which the Mellin representations are valid:

$$V(p^2l^2) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{v(\xi)}{\sin \pi\xi}[(m^2+p^2)l^2]^\xi \tag{13a}$$

$$V^{(1)}(p^2l^2) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{\xi v(\xi)}{\sin \pi\xi}[(p^2+m^2)l^2]^\xi \tag{13b}$$

$$V^{(2)}(p^2l^2) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{v(\xi)\xi(\xi-1)}{\sin \pi\xi}[(m^2+p^2)l^2]^\xi \tag{13c}$$

$$V^{(3)}(p^2l^2) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{v(\xi)\xi(\xi-1)(\xi-2)}{\sin \pi\xi}[(m^2+p^2)l^2]^\xi \tag{13d}$$

In this paper, we do not consider the explicit form of the entire analytic functions $V^{(i)}(p^2l^2)$ $(i=1,2,3)$, and therefore the concrete form of the dependence of $v(\xi)$ on the parameter ξ is not important. We only use the following properties:

$$v(0)=1, \qquad v'(0)=\lim \partial v(\xi)/\partial\xi, \qquad \lim v(\xi)=0$$
$$\xi\to 0, \qquad \xi\to -1 \tag{14}$$

For the passage from x space to p space in equations (11a)-(11e) we need calculations of the expressions $\delta S/\delta A_\mu$, $\delta S/\delta\varphi$, and $\delta S/\delta\varphi^*$. Taking into account the explicit form of S, the electromagnetic interaction action of charged particles, and after some simplifications, we have

$$\frac{\delta S}{\delta A_\nu(x)} = -\partial_\mu F_{\mu\nu}(x) - ie\varphi\partial_\nu\varphi^* + ie\varphi^*\partial_\nu\varphi + 2e^2A_\nu|\varphi|^2 \tag{15a}$$

$$\frac{\delta S}{\delta\varphi(x)} = (m^2-\Box)\varphi^* - ie\partial_\mu(A_\mu\varphi^*) - ieA_\mu\partial_\mu\varphi^* + e^2A_\mu^2(\varphi)^* \tag{15b}$$

$$\frac{\delta S}{\delta\varphi^*(x)} = (m^2-\Box)\varphi + ie\partial_\mu(A_\mu\varphi) + ieA_\mu\partial_\mu\varphi + e^2{A_\mu}^2\varphi \tag{15c}$$

With the expression (15a) the equation (11a) for the electromagnetic field takes the form, in the momentum representation,

$$\left\langle \int (dp)\left[-\left(\delta_{\mu\nu}p^2 - p_\nu p_\mu + \frac{1}{\alpha}p_\nu p_\mu\right)A_\mu(p)\right.\right.$$
$$+e\int (dp_1)(2p_1+p)_\nu \varphi^*(p_1)\varphi(p+p_1)$$
$$-2e^2\int\int (dp_1)(dp_2)A_\nu(p_1)\varphi(p-p_1+p_2)\varphi^*(p_2)$$
$$\left.\left.+K^A(p^2l^2)\frac{\delta}{\delta A_\nu(p)}\right]\frac{\delta F[A]}{\delta A_\nu(p)}\right\rangle = 0 \tag{16}$$

where the notation $(dp) = d^dp(2\pi)^{-d}$ has been used. If the explicit form of the gauge-invariant functional $F[A]$ is known, then from (16) we can define first, second, third terms, etc., for any order of the correlation function of the electromagnetic field. In particular, if $F[A] = A_{\nu_1}(q_1)$, then taking into account the identity

$$\delta_{\mu\nu} = (\delta_{\nu\rho} - p_\nu p_\rho/p^2 + \alpha p_\nu p_\rho/p^2)[\delta_{\rho\mu} - p_\rho p_\mu | p^2 + (1/\alpha)p_\mu p_\rho/p^2] \tag{17}$$

we have

$$\langle A_{\nu_1}(q_1)\rangle = -q_1^{-2}(\delta_{\nu_1\mu} - q_{1\nu_1}q_{1\mu}q_1^{-2} + \alpha q_1^{-2}q_{1\nu_1}q_{1\mu})$$
$$\times\left[e\int (dp_1)(2p_1+q_1)_\mu\langle\varphi(p_1+q_1)\varphi^*(p_1)\rangle + 2e^2\int\int (dp_1)(dp_2)\right.$$
$$\left.\times\langle A_\mu(p_1)\varphi^*(p_2)\varphi(q_1-p_1+p_2)\rangle\right] \tag{18}$$

Let $F[A] = A_{\nu_1}(q_1)A_{\nu_2}(q_2)$. Then, making use of (17) and integrating over (dp) from (16) it follows that

$$\langle A_{\nu_1}(q_1)A_{\nu_2}(q_2)\rangle$$
$$= [2\delta_{\nu_1\nu_2} - (q_1^{-2}q_{1\nu_1}q_{2\nu_2} + q_2^{-2}q_{2\nu_1}q_{2\nu_2})(1-\alpha)]\bar{\delta}^{(d)}(q_1+q_2)$$
$$\times K^A(q_1^2l^2)Q_2^0 - eQ_2^0\int (dp_1)\left[(2p_1+q_1)_\mu[\delta_{\nu_1\mu} - q_1^{-2}q_{1\mu}q_{1\nu_1}(1-\alpha)]\right.$$
$$\times\langle A_{\nu_2}(q_2)\varphi(p_1+q_1)\varphi^*(p_1)\rangle + (2p_1+q_2)_\mu[\delta_{\nu_2\mu} - q_2^{-2}q_{2\mu}q_{2\nu_2}(1-\alpha)]$$
$$\times\langle A_{\nu_1}(q_1)\varphi(q_2+p_1)\varphi^*(p_1)\rangle - 2e^2Q_2^0\int\int (dp_1)(dp_2)$$

$$\times[\delta_{\nu_1\mu}-q_{1\mu}q_{1\nu_1}q_1^{-2}(1-\alpha)]\langle A_\mu(p_1)A_{\nu_2}(q_2)\varphi(q_1-p_1+p_2)\varphi^*(p_2)\rangle$$

$$\left.+[\delta_{\nu_2\mu}-q_2^{-2}q_{2\mu}q_{2\nu_2}(1-\alpha)]\langle A_\mu(p_1)A_{\nu_1}(q_1)\varphi(q_2-p_1+p_2)\varphi^*(p_2)\rangle\right] \quad (19)$$

where $Q_2^0=(q_1^2+q_2^2)^{-1}$.

In the same manner, one can calculate three- and four-point correlation functions for the photon field. In the next section we show that a gauge-invariant quantity such as the photon polarization operator does not depend on the parameter α, terms depending on which mutually cancel. Therefore, it is convenient to the three- and four-point correlation functions of the photon for a concrete choice of the parameter α, namely $\alpha=1$. Thus, for the three-point photon correlation function we have

$$\begin{aligned}
&\langle A_{\nu_1}(q_1)A_{\nu_2}(q_2)A_{\nu_3}(q_3)\rangle\\
&\quad=2Q_3[K^A(q_1^2l^2)\delta_{\nu_1\nu_2}A_{\nu_3}(q_3)\bar\delta^d(q_1+q_3)+\text{cyclic perm}\,\{q_i\}]\\
&\quad-eQ_3\int(dp_1)[(2p_1+q_1)_{\nu_1}\langle\varphi(p_1+q_1)\varphi^*(p_1)A_{\nu_2}(q_2)\\
&\quad\times A_{\nu_3}(q_3)\rangle+\text{cyclic perm}\,\{q_i\}]\\
&\quad-2e^2Q_3\int\!\!\int(dp_1)(dp_2)[\langle\varphi^*(p_2)\varphi(q_1-p_1+p_2)\\
&\quad\times A_{\nu_1}(p_1)A_{\nu_2}(q_2)A_{\nu_3}(q_3)\rangle+\text{cyclic perm}\,\{q_i\}]
\end{aligned}\quad (20)$$

and for the fourth-point correlation function

$$\begin{aligned}
&\langle A_{\nu_1}(q_1)A_{\nu_2}(q_2)A_{\nu_3}(q_3)A_{\nu_4}(q_4)\rangle\\
&\quad=2Q_4[K^A(q_1^2l^2)\bar\delta^d(q_1+q_2)\delta_{\nu_1\nu_2}\\
&\qquad\times\langle A_{\nu_3}(q_3)A_{\nu_4}(q_4)\rangle+\text{cyclic perm}\,\{q_i\}]\\
&\qquad-eQ_4\int(dp_1)[(2p_1+q_1)_{\nu_1}\\
&\qquad\times\langle\varphi(p_1+q_1)\varphi^*(p_1)A_{\nu_2}(q_2)A_{\nu_3}(q_3)A_{\nu_4}(q_4)\rangle\\
&\qquad+\text{cyclic perm}\,\{q_i\}]-2e^2Q_4\\
&\qquad\times\int\!\!\int(dp_1)(dp_2)[\langle\varphi^*(p_2)\varphi(q_1-p_1+p_2)\\
&\qquad\times A_{\nu_1}(p_1)A_{\nu_2}(q_2)A_{\nu_3}(q_3)A_{\nu_4}(q_4)\rangle+\text{cyclic perm}\,\{q_i\}\}
\end{aligned}\quad (21)$$

where

$$Q_3=(q_1^2+q_2^2+q_3^2)^{-1},\qquad Q_4=(q_1^2+q_2^2+q_3^2+q_4^2)^{-1}$$

and

$$\bar\delta^{(d)}(p)=(2\pi)^d\delta^{(d)}(p)$$

Expressions (20) and (21) correspond to the correlation functions of the photon field, defined by the Schwinger-Dyson formalism. Now we consider correlation functions for scalar charged particles within this formalism. To calculate the two-point correlation function for a charged scalar particle, we pass to the momentum representation for equations (11b) and (11c) and choose $F[\varphi]=\varphi(q_1)\varphi^*(q_2)$. After an analogous procedure as followed above for the photon field, we get

$$\langle\varphi(q_1)\varphi^*(q_2)\rangle=2Q_2V(q_1^2l^2)\bar{\delta}^d(q_1+q_2)+eP+ePH+\frac{1}{\alpha}eN-e^2PP-e^2PPH-e^2PHH+e^3PPHH+e^3PHHH \tag{22}$$

where $Q_2=(2m^2+q_1^2+q_2^2)^{-1}$; and

$$P=Q_2\int(dp_1)[(p_1+q_1)_\mu\langle A_\mu(q_1-p_1)\varphi(p_1)\varphi^*(q_2)\rangle+(q_2+p_1)_\mu\langle A_\mu(p_1-q_2)\varphi^*(p_1)\varphi(q_1)\rangle] \tag{23a}$$

$$PH=l^2Q_2\int\int\int(dp)(dp_1)(dp_2)(2p_1+p_2)_\mu\times\langle A_\mu(p_2)H(p^2l^2)V^{(1)}(p_1^2l^2)[\bar{\delta}^d(p-q_1)\times\bar{\delta}^{(d)}(p_1-q_2)+\bar{\delta}^{(d)}(p-q_2)\bar{\delta}^{(d)}(p_1-q_1)]\bar{\delta}^{(d)}(p-p_1-p_2) \tag{23b}$$

$$N=Q_2\int(dp_1)[(p_1+q_1)_\mu\langle A_\mu(q_1-p_1)\varphi(p_1)\varphi^*(q_2)\rangle-(q_2+p_1)_\mu\langle A_\mu(p_1-q_2)\varphi(q_1)\varphi^*(p_1)\rangle] \tag{23c}$$

$$PP=Q_2\int\int(dp_1)(dp_2)[\langle A_\mu(p_2)A_\mu(q_1-p_1-p_2)\varphi(p_1)\varphi^*(q_2)\rangle+\langle A_\mu(p_2)A_\mu(q_2-p_1-p_2)\varphi(q_1)\varphi^*(p_1)\rangle] \tag{23d}$$

$$PPH=l^2Q_2\int\int\int(dp_1)(dp_2)(dp_3)(dp)\times\bar{\delta}^{(d)}(p-p_1-p_2-p_3)H(p^2l^2)V^{(1)}(p_3^2l^2)\times\langle A_\mu(p_1)A_\mu(p_2)\rangle[\bar{\delta}^{(d)}(p-q_1)\bar{\delta}^{(d)}(p_3-q_2)+\bar{\delta}^d(p-q_2)\bar{\delta}^d(p_3-q_1)] \tag{23e}$$

$$PHH = l^4 Q_2 \int (dp) \prod_{j=1}^{4} (dp_j) \bar{\delta}^{(d)}(p-p_1-p_2)\bar{\delta}^{(d)}$$
$$\times (p_2-p_3-p_4)\langle A_{\mu_1}(p_1)A_{\mu_2}(p_3)\rangle(2p_2+p_1)_{\mu_1}(2p_4+p_3)_{\mu_2}$$
$$\times[\bar{\delta}^d(p-q_1)\bar{\delta}^d(p_4-q_2)+\bar{\delta}^{(1)}(p-q_2)\bar{\delta}^d(p_4-q_1)]\tfrac{1}{6}$$
$$\times[V^{(2)}(p^2l^2)H(p_2^2l^2)H(p_4^2l^2)+H(p^2l^2)V^{(2)}(p_2^2l^2)H(p_4^2l^2)$$
$$+H(p^2l^2)H(p_4^2l^2)V^{(2)}(p_4^2l^2)] \tag{23f}$$

$$PPHH = l^4 Q_2 \int (dp) \prod_{j=1}^{5} (dp_j)$$
$$\times \bar{\delta}^{(d)}(p-p_1-p_2-p_3)\bar{\delta}^{(d)}(p_3-p_4-p_5)$$
$$\times(2p_5+p_4)_{\mu_2}\langle A_\mu(p_1)A_\mu(p_2)A_{\mu_2}(p_1)\rangle$$
$$+[\bar{\delta}^{(d)}(p-q_1)\bar{\delta}^{(d)}(p_5-q_2)\bar{\delta}^{(d)}(p-q_2)\bar{\delta}^{(d)}(p_5-q_1)]$$
$$\times\tfrac{1}{6}[V^{(2)}(p_5^2l^2)H(p_3^2l^2)H(p^2l^2)+H(p_5^2l^2)V^{(2)}(p_3^2l^2)H(p^2l^2)$$
$$+H(p_5^2l^2)H(p_3^2l^2)V^{(2)}(p^2l^2)] \tag{23g}$$

$$PHHH = l^6 Q_2 \int (dp) \prod_{j=1}^{6} (dp_j)\bar{\delta}^{(d)}(p-p_1-p_2)\bar{\delta}^{(d)}(p_1-p_3-p_4)$$
$$\times\bar{\delta}^{(d)}(p_3-p_5-p_6)(2p_1+p_2)_{\mu_1}$$
$$\times(2p_3+p_4)_{\mu_2}(2p_5+p_6)_{\mu_3}\langle A_{\mu_1}(p_2)A_{\mu_2}(p_4)$$
$$\times A_{\mu_3}(p_6)\rangle[\bar{\delta}^{(d)}(p-q_1)\bar{\delta}^{(d)}(p_5-q_2)+\bar{\delta}^{(d)}(p-q_2)\bar{\delta}^{(d)}(p_5-q_1)]$$
$$\times\tfrac{1}{24}\{H(p^2l^2)H(p_1^2l^2)[H(p_3^2l^2)V^{(3)}(p_5^2l^2)+H(p_5^2l^2)V^{(3)}(p_3^2l^2)]$$
$$+H(p_3^2l^2)H(p_5^2l^2)[H(p^2l^2)V^{(3)}(p_1^2l^2)$$
$$+H(p_1^2l^2)V^{(3)}(p^2l^2)]\} \tag{23h}$$

In these equations we have restricted ourselves to quantities of the order of e^3. The four-point function is calculated in an analogous manner. The result reads

$$\langle\varphi(q_1)\varphi^*(q_2)\varphi(q_3)\varphi^*(q_4)\rangle$$
$$= Q_4\left(2M+eR+eRH+\frac{1}{\alpha}eRA-e^2RR-e^2RRH-e^2RHH\right) \tag{24}$$

Here the following notations have been introduced:

$$Q_4=(4m^2+q_1^2+q_2^2+q_3^2+q_4^2)^{-1}$$
$$M=[V(q_1^2l^2)\bar{\delta}^{(d)}(q_1-q_2)\langle\varphi(q_3)\varphi^*(q_4)+\text{cyclic perm }\{q_i\}]$$

$$R=\int (dp_1)[(q_1+p_2)_\mu\langle A_\mu(q_1-p_1)\varphi(p_1)\varphi^*(q_2)\varphi(q_3)\varphi^*(q_4)\rangle$$
$$+(q_2+p_1)_\mu\langle A_\mu(p_1-q_2)\varphi(q_1)\varphi^*(p_1)\varphi(q_3)\varphi^*(q_4)\rangle$$
$$+\text{cyclic perm }\{q_j\}] \tag{25a}$$

$$RH=\int (dp)(dp_1)(dp_2)(2p_1+p_2)_\mu$$
$$\times\bar{\delta}^{(d)}(p-p_1-p_2)H(p^2l^2)V^{(1)}(p_1^2l^2)$$
$$\times[(\bar{\delta}^{(d)}(p_1-q_1)\bar{\delta}^{(d)}(p-q_2)+\bar{\delta}^{(d)}(p_1-q_2)\bar{\delta}^{d}(p-q_1))$$
$$\times\langle A_\mu(p_2)\varphi(q_3)\varphi^*(q_4)\rangle+\text{cyclic perm }\{q_j\}] \tag{25b}$$

$$RA=\int (dp_1)[(q_1+p_1)_\mu\langle A_\mu(q_1-p_1)\varphi(p_1)\varphi^*(q_2)\varphi(q_3)\varphi^*(q_4)\rangle$$
$$-(p_1+q_2)_\mu\langle A_\mu(p_1-q_2)\varphi(q_1)\varphi^*(p_1)\varphi(q_3)\varphi^*(q_4)\rangle$$
$$+\text{cyclic perm }\{q_j\}] \tag{25c}$$

$$RR=\int (dp_1)(dp_2)[\langle A_\mu(p_2)A_\mu(q_1-p_1-p_2)\varphi(p_1)\varphi^*(q_2)\varphi(q_3)\varphi^*(q_4)\rangle$$
$$+\langle A_\mu(p_2)A_\mu(p_1-q_2-p_2)\varphi(q_1)\varphi^*(p_1)\varphi(q_3)\varphi^*(q_4)\rangle$$
$$+\text{cyclic perm }\{q_j\}] \tag{25d}$$

$$RRH=l^2\int (dp)(dp_1)\cdots(dp_3)\bar{\delta}^{(d)}(p-p_1-p_2-p_3)H(p^2l^2)V^{(1)}(p_3^2l^2)$$
$$\times[(\bar{\delta}^{d}(p-q_1)\bar{\delta}^{(d)}(p_3-q_2)+\bar{\delta}^{(d)}(p-q_2)\bar{\delta}^{(d)}(p_3-q_1))$$
$$\times\langle A_\mu(p_1)A_\mu(p_2)\varphi(q_3)\varphi^*(q_4)\rangle+\text{cyclic perm }\{q_j\}] \tag{25e}$$

$$RHH=l^4\int (dp)\prod_{j=1}^{4}(dp_j)\bar{\delta}^{(d)}(p-p_1-p_2)$$
$$\times\bar{\delta}^{(d)}(p_2-p_3-p_4)(2p_2+p_1)_{\mu_1}$$
$$\times(2p_4+p_3)_{\mu_2}\tfrac{1}{6}[V^{(2)}(p^2l^2)H(p_2^2l^2)H(p_4^2l^2)$$
$$+H(p^2l^2)V^{(2)}(p_2^2l^2)H(p_4^2l^2)$$
$$\times H(p^2l^2)H(p_2^2l^2)V^{(2)}(p_4^2l^2)]$$
$$\times[(\bar{\delta}^{(d)}(p-q_1)\bar{\delta}^{d}(p_4-q_2)+\bar{\delta}^{(d)}(p-q_2)$$
$$\times\bar{\delta}^{(d)}(p_4-q_1))\langle A_{\mu_1}(p_1)A_{\mu_2}(p_2)\varphi(q_3)\varphi^*(q_4)\rangle$$
$$+\text{cyclic perm }\{q_j\}] \tag{25f}$$

Here we have restricted ourselves to order e^2, which is sufficient for further concrete calculations. In (25a)-(25f) cyclic permitation over (q_j) is understood as $q_1 \leftrightarrow q_3$ and $q_2 \leftrightarrow q_4$.

4. THE VACUUM POLARIZATION DIAGRAM

Now we consider concrete processes within the Schwinger-Dyson formalism. First, we calculate the process with two external photon lines. In our scheme the vacuum polarization is given by the two-point correlation function of the photon which is represented in (19). Let us consider each term in (19) separately. The first term in (19), i.e., for $e=0$, defines the modified propagator of the photon field:

$$\langle A_{\nu_1}(q_1)A_{\nu_2}(q_2)\rangle = [\delta_{\nu_1\nu_2} - \bar{q}_1^2 q_{1\nu_1}q_{2\nu_2}(1-\alpha)]\bar{q}_1^2 K^A(q_1^2 l^2)\bar{\delta}^{(d)}(q_1+q_2) \quad (26)$$

Similarly, from (22) for $e=0$ we determine the modified propagator of the scalar particle:

$$\langle \varphi(q_1)\varphi^*(q_2)\rangle = (m^2+q_1^2)^{-1}\bar{\delta}^{(d)}(q_1+q_2)\,V(q_1^2 l^2) \quad (27)$$

From equation (19) for the two-point correlation photon function we rewrite those terms which are proportional to e^2:

$$\begin{aligned}
&\langle A_{\nu_1}(q_1)A_{\nu_2}(q_2)\rangle^{(1)} \\
&= -2e^2 Q_2^0 \int\int (dp_1)(dp_2)\{[\delta_{\nu_1\mu} - \bar{q}_1^2 q_{1\mu}q_{1\nu_1}(1-\alpha)] \\
&\quad \times \langle A_\mu(p_1)A_{\nu_2}(q_2)\varphi(q_2-p_1+p_2)\varphi^*(p_2)\rangle + [\delta_{\nu_2\mu} - \bar{q}_2^2 q_{2\mu}q_{2\nu_2}(1-\alpha)] \\
&\quad \times \langle A_\mu(p_1)A_{\nu_1}(q_1)\varphi(q_2-p_1+p_2)\varphi^*(p_2)\rangle\}
\end{aligned}$$

Substituting quantities (26) and (27) for the modified propagators of the photon and the scalar particle and also integrating over (dp_1), we get

$$\begin{aligned}
&\langle A_{\nu_1}(q_1)A_{\nu_2}(q_2)^{(1)} \\
&= 2Q_2^0\bar{\delta}^{(d)}(q_1+q_2)[\delta_{\nu_1\mu} - \bar{q}_1^2 q_{1\mu}q_{2\nu_1}(1-\alpha)] \\
&\quad \times \bar{q}_1^2 K^A(q_1^2 l^2)\Pi^{(1)}_{\mu\nu}(q_1)[\delta_{\nu\nu_2} - \bar{q}_2^2 q_{2\nu}q_{2\nu_2}(1-\alpha)]
\end{aligned}$$

where

$$\Pi^{(1)}_{\mu\nu}(q) = -2e^2\delta_{\mu\nu}\int (dp)\,V(p^2 l^2)(m^2+p^2)^{-1} \quad (28)$$

Further, we employ Feynman-type diagrammic rules for the stochastic quantization scheme [for details, see Bern *et al.* (1987*a*) and Dineykhan and Namsrai (1988)] and represent the corresponding diagram in Figure

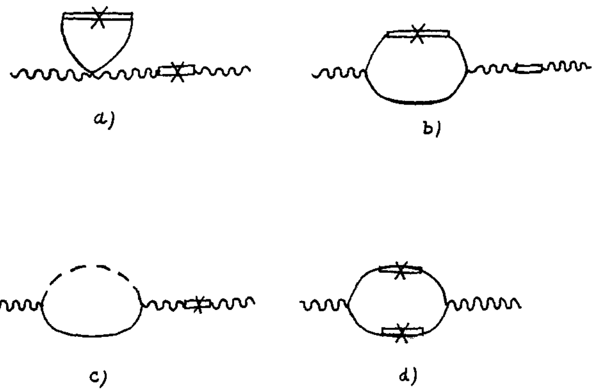

Fig. 1. Vacuum polarization diagram.

1a. Making use of the Mellin representation (13a) for $V(p^2l^2)$ and integrating over (dp), we obtain from (28)

$$\Pi^{(1)}_{\mu\nu}(q) = -2e^2(16\pi^2)^{-1}\delta_{\mu\nu}\frac{m^2}{2i}$$
$$\times\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi\frac{v(\xi)}{\sin\pi\xi}\frac{\Gamma(-1-\xi)}{\Gamma(1-\xi)}(m^2l^2)^{\xi}, \qquad -1\le\beta\le 2$$

Taking into account (14) and carrying out contour integration at the points $\xi=-1$, $\xi=0$ and keeping terms to lower order of l, we have

$$\Pi^{(1)}_{\mu\nu}(q) = -e^2\delta_{\mu\nu}(8\pi^2)^{-1}\{\sigma\bar{l}^2+m[v'(0)+\ln m^2l^2-1]\} \tag{29}$$

where

$$\sigma = \lim_{\xi\to-1} v(\xi)/(1+\xi) \tag{30}$$

Now let us consider the other term in (19), which has the form

$$\langle A_{\nu_1}(q_1)A_{\nu_2}(q_2)\rangle$$
$$= -eQ_2^0\int (dp_1)\{(2p_1+q_1)_\mu[\delta_{\nu_1\beta}-\bar{q}_1^2q_{1\mu}q_{1\nu_1}(1-\alpha)]$$
$$\times\langle A_{\nu_2}(q_2)\varphi(p_1+q_1)\varphi^*(p_1)\rangle+(2p_1+q_2)_\mu(\delta_{\nu_2\mu}-\bar{q}_2^2q_{2\mu}q_{2\nu_2}$$
$$+\alpha\bar{q}_2^2q_{2\mu}q_{2\nu_2})\langle A_{\nu_1}(q_1)\varphi(p_1+q_2)\varphi^*(p_1)\rangle \tag{31}$$

From (31) it is seen that for the description of the polarization diagram one needs to express the correlation function of the photon or scalar particle through high orders of e. Finding $\varphi(p_1+q_j)\varphi^*(p_1)$ from (22) and (23a) and substituting it into (31), we get

$$\langle A_{\nu_1}(q_1)A_{\nu_2}(q_2)\rangle^{(2)}$$

$$= -e^2 Q_2^0 \int (dp_1)\Big\{[\delta_{\nu_1\mu} - \bar{q}_1^2 q_{1\mu}q_{1\nu_1}(1-\alpha)]$$

$$\times(2p_1+q_1)_\mu[2m^2+p_1^2+(p_1+q_1)^2]^{-1}\int (dp_2)[(p_1+p_2+q_1)_\nu$$

$$\times\langle A_{\nu_2}(q_2)A_\nu(p_1+q_1-p_2)\varphi(p_2)\varphi^*(p_1)\rangle+(p_1+p_2)_\nu\langle A_{\nu_2}(q_2)A_\nu(p_2-q_1)$$

$$\times\varphi(p_1+q_1)\varphi^*(p_2)\rangle]+\text{cyclic perm}\,(q_1\leftrightarrow q_2)\Big\}$$

Next, taking into account (26) and (27) and integrating over (dp_1), we get

$$\langle A_{\nu_1}(q_1)A_{\nu_2}(q_2)\rangle^{(2)}$$
$$=2\bar{\delta}^d(q_1+q_2)Q_2[\delta_{\nu_1\mu}-\bar{q}_1^2 q_{1\mu}q_{1\nu_1}(1-\alpha)]\bar{q}_1^2 K^A(q_1^2 l^2)$$
$$\times\Pi^{(2)}_{\mu\nu}(q_1)[\delta_{\nu\nu_2}-\bar{q}_2^2 q_{2\nu}q_{2\nu_2}(1-\alpha)]$$

where

$$\Pi^{(2)}_{\mu\nu}(q) = -e^2\int (dp)\frac{(2p+q)_\nu(2p+q)_\mu}{2m^2+p^2+(p+q)^2}\left[\frac{V(p^2l^2)}{m^2+p^2}+\frac{V((p+q)^2l^2)}{m^2+(p+q)^2}\right]$$

The corresponding diagram is sketched in Figure 1b. After some simple calculations as done above, we have the following expression for $\Pi^{(2)}_{\mu\nu}(q)$:

$$\Pi^{(2)}_{\mu\nu}(q) = e^2(16\pi^2)^{-1}\Big(-l^2\delta_{\mu\nu}\sigma - 2m^2\delta_{\mu\nu}(v'(0)+\ln l^2m^2)$$

$$-\tfrac{1}{3}q^2\delta_{\mu\nu}+(q_\nu q_\mu - q^2\delta_{\mu\nu})\int_0^1 dx(1-x)^2\Big\{v'(0)+\ln l^2m^2$$

$$+\ln\left[1+\frac{q^2}{m^2}\frac{x}{2}(1-\tfrac{1}{2}x)\right]-\ln(1-x)\Big\}\Big) \qquad (32)$$

where σ is given by (30).

Further, define the quantity $\varphi(p_1+q_1)\varphi^*(p_1)$ by (22) and (23b) and substitute it into (31). The result reads

$$\langle A_{\nu_1}(q_1)A_{\nu_2}(q_2)\rangle^{(3)}$$

$$= -e^2Q_2l^2\int (dp_1)\Big\{[\delta_{\nu_1\mu}-\bar{q}_1^2 q_{1\mu}q_{1\nu_1}(1-\alpha)]$$

$$\times(2p_1+q_1)_\mu[2m^2+p_1^2+(p_1+q_1)^2]^{-1}\int (dp)(dp_3)(dp_2)(2p_3+p_2)_\nu$$

$$\times\langle A_\nu(p_2)A_{\nu_2}(q_2)\rangle H(p^2l^2)V^{(1)}(p_3^2l^2)[\bar{\delta}^d(p-q_1-p_1)\bar{\delta}^d(p_3-p_1)$$

$$+\bar{\delta}^d(p-p_1)\bar{\delta}^d(p_3-p_1-q_1)]\bar{\delta}^d(p-p_3-p_2)+\text{cyclic perm}\,(q_1\leftrightarrow q_2)\}$$

Making use of (26) and carrying out integration over (dp), (dp_2), and (dp_3), we get

$$\langle A_{\nu_1}(p_1)A_{\nu_2}(q_2)\rangle^{(3)}$$
$$=2Q_2^0\bar{\delta}^d(q_1+q_2)[\delta_{\nu_1\mu}-\bar{q}_1^2q_\mu q_{1\nu_1}(1-\alpha)]\bar{q}_1^2K^A(q_1^2l^2)$$
$$\times\Pi_{\mu\nu}^{(3)}(q_1)[\delta_{\nu\nu_2}-\bar{q}_2^2q_{2\mu}q_{2\nu_2}(1-\alpha)]$$

where

$$\Pi_{\mu\nu}^{(3)}(q)=-e^2\int(dp)(2p+q)_\nu(2p+q)_\mu[2m^2+p^2+(p+q)^2]^{-1}$$
$$\times\{V^{(1)}(p^2l^2)/[m^2+(p+q)^2]+V^{(1)}((p+q)^2l^2)/(m^2+p^2)\}$$

The corresponding diagram is shown in Figure 1c. After some simple calculations we have finally

$$\Pi_{\mu\nu}^{(3)}(q)=e^2(16\pi^2)^{-1}[\sigma\bar{l}^2\delta_{\mu\nu}-2m^2\delta_{\mu\nu}-\tfrac{1}{3}(q^2\delta_{\mu\nu}-q_\nu q_\mu)] \qquad (33)$$

Now we go to the next term. The expression for $\varphi(p_1+q_1)\varphi^*(p_1)$ is obtained from (22) and (23c) after substituting their values into (31). Thus, in this approximation we have

$$\langle A_{\nu_1}(q_1)A_{\nu_2}(q_2)\rangle^{(4)}$$
$$=-e^2Q_2^0\frac{1}{\alpha}\int(dp_1)\Big\{[\delta_{\nu_1\mu}-\bar{q}_1^2q_{1\mu}q_{1\nu_1}(1-\alpha)]$$
$$\times(2p_1+q_1)_\mu[2m^2+p_1^2+(p_1+q_1)^2]^{-1}\int(dp_2)[(p_1+p_2+q_1)_\nu$$
$$\times\langle A_{\nu_2}(q_2)A_\nu(p_1+q_1-p_2)\varphi(p_2)\varphi^*(p_1)\rangle-(p_2+p_1)_\nu$$
$$\times\langle A_{\nu_2}(q_2)A_\nu(p_2-p_1)\varphi(p_1+q_1)\varphi^*(p_2)\rangle]+\text{cyclic perm }(q_1\leftrightarrow q_2)\Big\}$$

With the expressions (26) and (27) integration over (dp_2) gives

$$\langle A_{\nu_1}(q_1)A_{\nu_2}(q_2)\rangle^{(4)}$$
$$=2Q_2^0\delta^d(q_1+q_2)[\delta_{\nu_1\mu}-\bar{q}_1^2q_{1\mu}q_{1\nu_1}(1-\alpha)]\bar{q}_1^2K^A(q_1^2l^2)$$
$$\times\Pi_{\mu\nu}^{(4)}(q_1)[\delta_{\nu\nu_2}-\bar{q}_2^2\dot{q}_{2\nu_2}q_{2\nu}(1-\alpha)]$$

where

$$\Pi_{\mu\nu}^{(4)}(q)=-\frac{1}{\alpha}e^2\int(dp)[2m^2+p^2+(p+q)^2]^{-1}(2p+q)_\nu(2p+q)_\mu$$
$$\times\{V(p^2l^2)/(m^2+p^2)-V((p+q)^2l^2)/[m^2+(p+q)^2]\} \qquad (34)$$

In the second term of this expression, changing the integration variable $p \to p' = p - q$ and taking into account its integration property, it is easy to verify that $\Pi^{(4)}_{\mu\nu}(q)$ is identically equal to zero, i.e., the gauge-invariant quantity $\Pi_{\mu\nu}(q)$ does not depend on the gauge-fixing parameter α.

Finally, substituting the expression (18) for $A_{\nu_j}(q_j)$ into (31) and taking into account (24) and (27) and after some simplifications, we get from (31)

$$\begin{aligned}&\langle A_{\nu_1}(q_1)A_{\nu_2}(q_2)\rangle^{(5)}\\&\quad = 2Q_2^0\bar{\delta}^d(q_1+q_2)[\delta_{\nu_1\mu} - \bar{q}_1^2 q_{1\nu_1}q_{1\mu}(1-\alpha)]\\&\qquad \times \bar{q}_1^2\Pi^{(5)}_{\mu\nu}(q_1^2)[\delta_{\nu_1\nu_2} - \bar{q}_2^2 q_{2\nu_2}q_{2\nu}(1-\alpha)]\end{aligned}$$

where

$$\begin{aligned}\Pi^{(5)}_{\mu\nu}(q) = e^1 \int (dp)(2p+q)_\nu(2p+q)_\mu V(p^2l^2)\,V((p+q)^2l^2)\\ \times\{(m^2+p^2)[m^2+(p+q)^2]\}^{-1}\end{aligned}$$

and the corresponding diagram is shown in Figure 1d. A simple calculation for $\Pi^{(5)}_{\mu\nu}$ gives

$$\begin{aligned}\Pi^{(5)}_{\mu\nu}(q) &= (e^2/16\pi^2)(2\sigma\bar{l}^2\delta_{\mu\nu} + 4m^2\delta_{\mu\nu}(v'(0) + \ln m^2l^2) + \tfrac{1}{3}q^2\delta_{\mu\nu}\\&\quad + 2(q^2\delta_{\mu\nu} - q_\nu q_\mu)\int_0^1 dx(1-2x)^2\\&\quad \times\left[v'(0) + \ln m^2l^2 + \ln\left(1 + \frac{q^2}{m^2}x(1-x)\right) - \ln(1-x)\right]\end{aligned} \qquad (35)$$

The sum of all terms (29) and (32)-(34) gives the full corrections due to the diagrams shown in Figures 1a-1d to the vacuum polarization of the photon field:

$$\Pi_{\mu\nu}(q) = \sum_{i=1}^{5} \Pi^{(j)}_{\mu\nu}(q) = e^2(\delta_{\mu\nu}q^2 - q_\nu q_\mu)\Pi(q^2) \qquad (36)$$

where

$$\begin{aligned}\Pi(q^2) = \frac{1}{16\pi^2}\int_0^1 dx\Big\{&[2(1-2x)^2 - (1-x)^2][v'(0) + \ln m^2l^2 - \ln(1-x)]\\&+ 2(1-2x)^2\ln\left(1 + \frac{q^2}{m^2}x(1-x)\right) - (1-x)^2\\&\times\ln\left[1 + \frac{1}{2}\frac{q^2}{m^2}x\left(1 - \frac{x}{2}\right)\right] - \frac{1}{3}\Big\}\end{aligned}$$

From the explicit form (36) it is easily seen that the gauge-invariant condition for the vacuum polarization diagram is fulfilled automatically and the obtained expression agrees with the usual result of nonlocal quantum electrodynamics (Efimov, 1977) and a dependence on the gauge-fixing parameter does not appear here.

5. SELF-ENERGY DIAGRAMS FOR A SCALAR PARTICLE

Now, let us consider the self-energy diagrams of a scalar particle. In our scheme, these diagrams are defined by the two-point correlation function represented in (22). In the preceding section we have shown that the dependence on the gauge-fixing parameter α does not appear in calculations of gauge-invariant quantities such as the vacuum polarization of the photon field. Therefore, for calculation purpose, we further take $\alpha=1$ for the modified photon propagator. On the other hand, from expressions (22) and (24) for two- and three-point correlation functions of the scalar particle it is seen that this dependence on α also falls out. First, from (22) we separate terms which are proportional to e^2. Taking into account (23), we get from (22)

$$\begin{aligned}&\langle\varphi(q_1)\varphi^*(q_2)\rangle^{(1)}\\&=-e^2Q_2\int\int(dp_1)(dp_2)[\langle A_\mu(p_2)A_\mu(q_1-p_1-p_2)\\&\quad\times\varphi(p_1)\varphi^*(q_2)\rangle+\langle A_\mu(p_2)A_\mu(q_2-p_1-p_2)\varphi(q_1)\varphi^*(p_2)\rangle]\end{aligned}\tag{36}$$

Further, making use of (26) and (27) and integrating over (dp_j), we have

$$\langle\varphi(q_1)\varphi^*(q_2)\rangle^{(1)}=-8e^2Q_2[V(q_1^2l^2)/(m^2+q_1^2)]\bar{\delta}^d(q_1-q_2)\int dp\,K^A(p^2l^2)/p^2$$

The corresponding diagram is shown in Figure 2a. Turning to the Mellin representation (12) for $K^A(p^2l^2)$ and carrying out the integration over (dp) and restricting consideration to lower orders of l, we obtain after calculations of the contour integral

$$\langle\varphi(q_1)\varphi^*(q_2)\rangle^{(1)}=-\frac{e^2}{2\pi^2}\bar{\delta}^d(q_1-q_2)Q_2\frac{V(q_1^2l^2)}{m^2+q_1^2}\frac{\sigma}{l^2}\tag{37}$$

Now, we express $\varphi(q_1)\varphi^*(q_2)$ through (23d) and carry out some simple calculations for (22). The result reads

$$\begin{aligned}\langle\varphi(q_1)\varphi^*(q_2)\rangle^{(2)}&=-8e^2l^2Q^2V^{(1)}(q_1^2l^2)H(q_1^2l^2)\delta^d(q_1-q_2)\\&\quad\times\int(dp)K^A(p^2l^2)\bar{p}^2\end{aligned}$$

The corresponding diagram is sketched in Figure 2b.

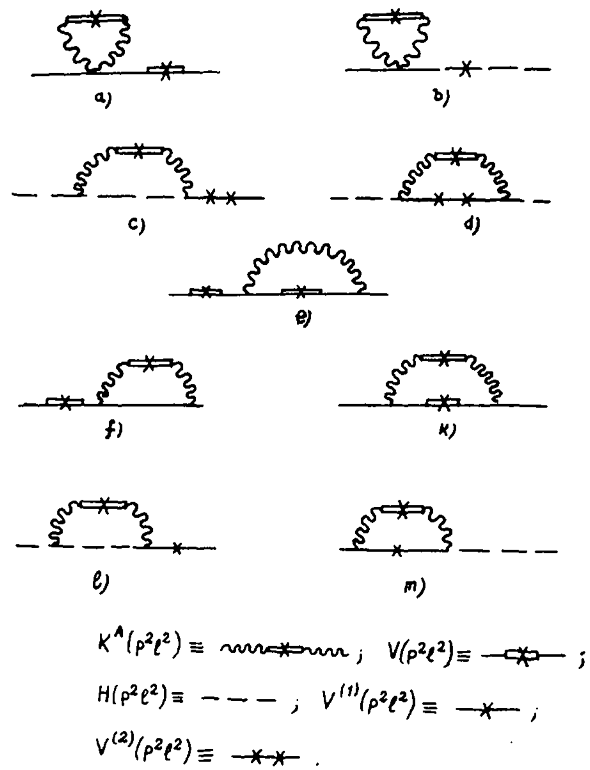

Fig. 2. The self-energy diagrams of the scalar particle.

After analogous calculations as done above, we have

$$\langle\varphi(q_1)\varphi^*(q_2)\rangle^{(2)} = -\frac{e^2}{2\pi^2}\frac{\bar{\delta}^d(q_1-q_2)}{2m^2+q_1^2+q_2^2}\frac{V^{(1)}(q_1^2l^2)}{m^2+q_1^2}\frac{\sigma}{l^2} \tag{38}$$

From the expression (22) we see that one term proportional to e^2 remains. For the calculation of this term, we take (23e) and simplify (22). Then

$$\begin{aligned}\langle\varphi(q_1)\varphi^*(q_2)\rangle^{(3)}\\ &= -\tfrac{2}{3}e^2l^4Q_2V^{(2)}(q_1^2l^2)H(q_1^2l^2)\int(dp)\bar{\delta}^d(q_1-q_2)(p+2q_1)^2\\ &\quad\times H((p+q_1)^2l^2)K^A(p^2l^2)\bar{p}^2-\tfrac{1}{3}e^2l^4Q_2H(q_1^2l^2)\int(dp)\\ &\quad\times(p+2q_1)^2H((p+q_1)^2l^2)V^{(2)}((p+q_1)^2l^2)\bar{p}^2K^A(p^2l^2)\bar{\delta}^d(q_1-q_2)\end{aligned}$$

The corresponding diagram is shown in Figure 2c. In order to present an explicit form of this expression, we carry out some standard simple calculations, which are reduced to the following formula:

$$\begin{aligned}\langle\varphi(q_1)\varphi^*(q_2)\rangle^{(3)}\\ &= -\frac{2}{3}\frac{e^2}{16\pi^2}\bar{\delta}^d(q_1-q_2)Q_2\frac{V^{(2)}(q_1^2l^2)}{m^2+q_1^2}\\ &\times\left[\sigma\bar{l}^2+m^2[v'(0)+\ln m^2l^2]-2q_1^2\left(v'(0)+\ln m^2l^2-\frac{11}{12}\right)\right]\\ &-\frac{1}{3}\frac{e^2}{16\pi^2}\bar{\delta}^d(q_1-q_2)Q_2\frac{1}{m^2+q_1^2}(2\sigma\bar{l}^2-m^2+2q_1^2)\end{aligned} \tag{39}$$

Now let us consider other terms which are proportional to e. From expression (22) it is easily seen that there exist three such terms. One of them is represented in (23c), which does not give a contribution to the self-energy diagram of the scalar particle as in the case of the polarization diagram. The other expression is given in (23b). To obtain terms proportional to e^2, $A_\mu(q_1)$ should be expressed through the quantity represented in (18). Then the corresponding diagram is sketched in Figure 2d, which is proportional to the integral

$$\int (dp)p_\mu f(p^2)$$

Therefore, it does not contribute to the self-energy diagram. Finally, we consider the expression presented in (23a):

$$\begin{aligned}\langle\varphi(q_1)\varphi^*(q_2)\rangle = eQ_2\int (dp_1)[(p_1+q_1)_\mu\langle A_\mu(q_1-p_1)\varphi(p_1)\varphi^*(q_2)\rangle\\ +(q_2+p_1)_\mu\langle A_\mu(p_1-q_2)\varphi(q_1)\varphi^*(p_1)\rangle]\end{aligned} \tag{40}$$

From this expression it is seen that to obtain terms proportional to e^2 we need to express $A_\mu(q)$ or $\varphi(p_1)\varphi^*(q_2)$ through values of high order in e.

First, we define $A_\mu(q_1-p_1)$ through the expression given by (18) and carry out some standard calculations from (40). The result reads

$$\begin{aligned}\langle\varphi(q_1)\varphi^*(q_2)\rangle^{(4)}\\ = -2e^2Q_2\frac{V(q_1^2l^2)}{m^2+q_2^2}\bar{\delta}^d(q_1-q_2)\int (dp)\frac{(p+2q_1)^2}{p^2}\frac{V((p+q_1)^2l^2)}{m^2+(p+q_1)^2}\end{aligned}$$

The corresponding diagram is presented in Figure 2e. Next, making use of the Mellin representation (13) for $V(p^2l^2)$ and of the Feynman parametrization, we obtain after integration

$$\langle\varphi(q_1)\varphi^*(q_2)\rangle^{(4)}$$
$$=-\frac{e^2}{16\pi^2}\frac{V(q_1^2l^2)}{(m^2+q_1^2)^2}\bar{\delta}^d(q_1-q_2)$$
$$\times\{\sigma\bar{l}^2+m^2[v'(0)+\ln m^2l^2-1]-2q_1^2[v'(0)+\ln m^2l^2-\tfrac{1}{3}]\} \quad (41)$$

Now let us express $\varphi(p_1)\varphi^*(q_2)$ through the quantity given by (23a) and (23b). For this, we substitute (23a) with (40) and carry out some elementary calculations. Thus,

$$\langle\varphi(q_1)\varphi^*(q_2)\rangle^{(5)}$$
$$=2e^2Q_2\bar{\delta}^d(q_1-q_2)\frac{V(q_1^2l^2)}{m^2+q_1^2}\int(dp)\frac{(p+2q_1)^2}{2m^2+(p+q_1)^2+q_2^2}$$
$$\times\frac{K^A(p^2l^2)}{p^2}+2e^2Q_2\bar{\delta}^d(q_1-q_2)$$
$$\times\int(dp)\frac{(p+2q_1)^2}{2m^2+(p+q_1)^2+q_2^2}\frac{K^A(p^2l^2)}{p^2}\frac{V((p+q_1)^2l^2)}{m^2+(p+q_1)^2}$$
$$\approx\frac{e^2V(q_2^2l^2)}{(m^2+q_2^2)^2}\frac{\bar{\delta}^d(q_1-q_2)}{16\pi^2}$$
$$\times\{\sigma l^{-2}+2m^2[v'(0)+\ln m^2l^2+\ln 2]$$
$$-q_1^2[v'(0)+\ln m^2l^2+\ln 2-\tfrac{5}{2}]\}-\frac{e^2}{8\pi^2}\frac{\bar{\delta}^d(q_1-q_2)}{(m^2+q_1^2)^2}$$
$$\times\{m^2[v'(0)+\ln m^2l^2+2\ln 2]+q_1^2[v'(0)$$
$$+\ln m^2l^2+1+\tfrac{5}{3}\ln 2]\}+O(l^2) \quad (42)$$

The corresponding diagrams are shown in Figures 2f–2k. Finally, substitution (23b) into (40) and analogous simplification give the result

$$\langle\varphi(q_1)\varphi^*(q_2)\rangle^{(6)}$$
$$=\frac{2e^2l^2}{2m^2+q_1^2+q_2^2}V^{(1)}(q_1^2l^2)\bar{\delta}^d(q_1-q_2)\int(dp)\frac{K^A(p^2l^2)}{p^2}$$
$$\times\frac{(p+2q_1)^2}{2m^2+(p+q_1)^2+q_2^2}H((p+q_1)^2l^2)$$
$$+\frac{2e^2l^2}{2m^2+q_1^2+q_2^2}H(q_2^2l^2)\bar{\delta}^d(q_1-q_2)$$
$$\times\int(dp)\frac{K^A(p^2l^2)}{p^2}\frac{(p+2q_1)^2}{2m^2+(p+q_1)^2+q_2^2}V^{(1)}((p+q_1)^2l^2$$

In the limit of the $O(l^2)$ term we have

$$\langle\varphi(q_1)\varphi^*(q_2)\rangle^{(6)}$$

$$= -\frac{e^2\bar{\delta}^d(q_1-q_2)}{16\pi^2}\frac{V^{(1)}(q_1^2l^2)}{(m^2+q_1^2)^2}\{m^2[v'(0)+\ln l^2m^2+2\ln 2]$$

$$+q^2[v'(0)+\ln m^2l^2+1+\tfrac{4}{3}\ln 2]\}$$

$$+\frac{e^2}{16\pi^2}\frac{\bar{\delta}^d(q_1-q_2)}{(m^2+q_1^2)^2}\left(-\frac{\sigma}{l^2}+m^2-2q_1^2\right)+O(l^2) \tag{43}$$

The corresponding diagrams are given in Figures 2l–2m. Assuming that the elementary length l is small, and momentum q is also not high, i.e., $m^2l^2 \ll 1$ and $q^2/m^2<1$, then the sum of expressions (37)–(39) and (41)–(43) gives the following contribution to the two-point correlation function of a scalar particle:

$$\langle\varphi(q_1)\varphi^*(q_2)\rangle=\frac{\bar{\delta}^d(q_1-q_2)}{(m^2+q_1^2)^2}\Sigma(q_1)$$

where $\Sigma(q)$ gives the contribution to self-energy diagrams of the scalar particle

$$\Sigma(q)=-\frac{5e^2}{16\pi^2}\frac{\sigma}{l^2} \tag{44}$$

where σ is defined by (30).

By means of (44) it is easy to calculate the correction to the mass value of the scalar particle:

$$\delta m^2=m_0^2-m^2=-\Sigma(m^2)=\frac{5\alpha}{4\pi}\frac{\sigma}{l^2} \tag{45}$$

Here $\alpha=e^2/4\pi\approx 1/137$.

Expression (45) agrees with the usual result of nonlocal scalar electrodynamics (Efimov, 1977).

ACKNOWLEDGMENTS

M.D. would like to thank the IREX at Princeton, New Jersey, and the High Energy Theory Group of the Physics Department, Syracuse University, Syracuse, New York, especially Prof. K. C. Wali and C. Graves, for their kind hospitality during a visit in the United States.

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